A LEAST ABSOLUTE BOUND APPROACH TO ICA

APPLICATION TO THE MLSP 2006 COMPETITION

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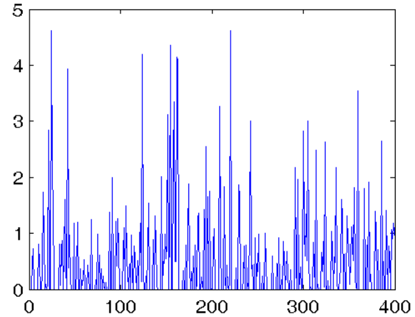
Independent Component Analysis

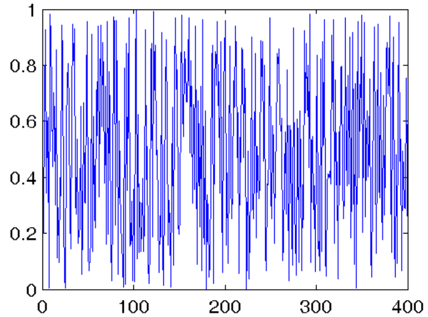
- Mixture model (linear and instantaneous)
  - $\mathbf{x}(\mathbf{t}) = \mathbf{A}\mathbf{s}(\mathbf{t})$ (n sources, n mixtures, 1≤t≤N)
- Aim: identify blindly the mixing coefficients (t omitted)
  - $\mathbf{y} = \mathbf{B}\mathbf{x} = \mathbf{B}\mathbf{A}\mathbf{s} = \mathbf{C}\mathbf{s}$ s.t. $\mathbf{C} = \mathbf{P}\mathbf{D}$ (permutation & scaling)
- Main assumption
  - Elements of $\mathbf{s}$ are statistically independent
- 2-stage separation
  - Whitening using SVD: $\mathbf{z} = \mathbf{V}(\mathbf{x} - \langle\mathbf{x}\rangle)$ s.t. $\mathrm{E}(\mathbf{z}\mathbf{z}^{\mathbf{T}}) = \mathbf{I}$
  - Unmixing: $\mathbf{y} = \mathbf{W}\mathbf{z}$ s.t. $\mathbf{W}\mathbf{W}^{\mathbf{T}} = \mathbf{I}$
- Perf. Index of the Competition

$$\mathrm{SIR} = \frac{1}{n}\sum_{i=1}^{n} 10\log_{10}\frac{\max_j c_{ij}^2}{\sum_j c_{ij}^2 - \max_j c_{ij}^2}$$

2006 MLSP Competition : BSS contest

- Sources: positive, 2 types (50%-50%)
  - Sparse in [0,∞[(super-Gaussian)
  - Uniform in [0,1] (subGaussian)





- 4 sub-problems to solve:

  (for each: 90% of the SIRs from MC runs > 15dB)

  - Large-scale problem (N=5000, largest n, **A** : rand.)
  - Small sample problem (lowest N, n=50, **A** : rand.)
  - Ill-conditioned mixtures (N=5000, largest n, **A** : Hilbert x Givens)
  - Noisy mixtures (N=5000, n=50, **A** : rand., largest σ_{noise})

To fit or not to fit (the method to the data)

- Two main approaches
  - Use general-purpose algorithms (Jade, fastICA,...)
  - Use *dedicated algorithms*, that better match to the a priori knowledge on the sources (Positive, Bounded Srcs). Here: *one-side bounded* sources: let's deal with that knowledge !
- Outline:
  - Ad-hoc criterion *fitted to the sources*
  - Optimization algorithm *fitted to the criterion*
  - Results on MLSP 06 benchmark

Minimum Range approach to ICA

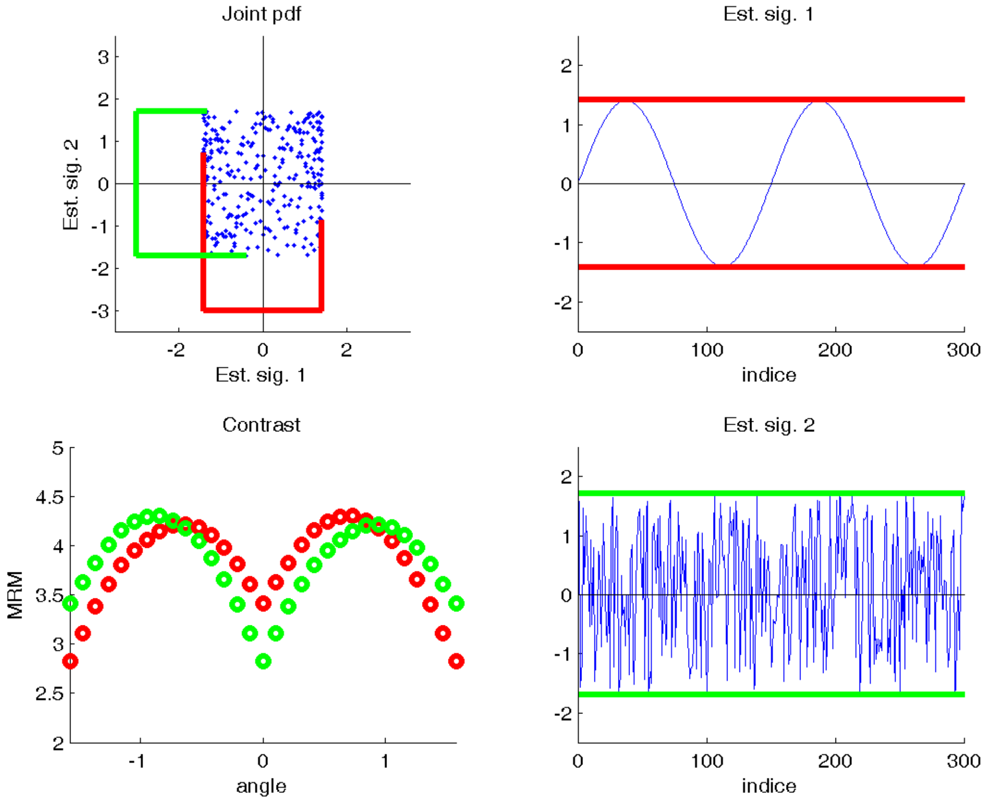
- Previous work about double-bounded sources…

  [Vrins, Jutten & Verleysen, ICASSP 2005]

- Toy illustration:
  - Sine
  - Uniform

<u>Issue #1:</u>
MLSP Contest sources are double-bounded and single-bounded !!!

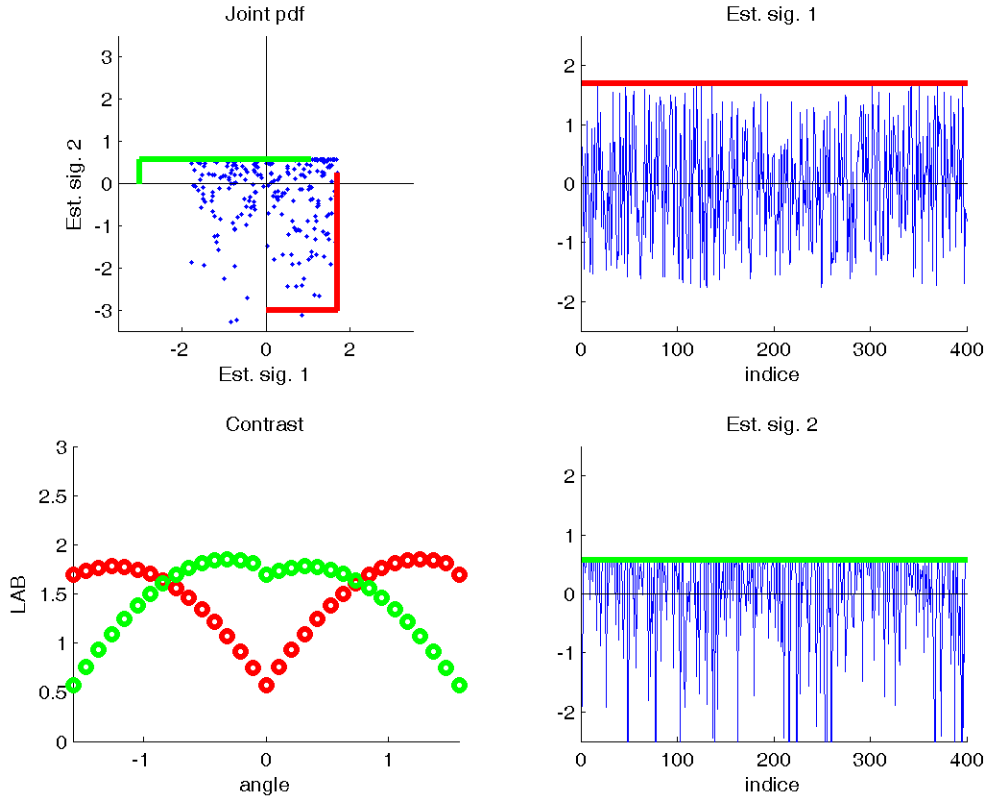
<u>Issue #2:</u>
seeked points are not stationary !!!



Issue #1: Least Absolute Bound ICA

- Idea: measure the range on one side only

- Toy illustration:
  - MLSP sources



Issue #1: Solved ☺
OK for double- and single-bounded sources

Issue #2: Pending… ☹
LAB is not everywhere differentiable either

R vs LAB: estimation of extreme pts

- Define
  - $\mathbf{y}' = \text{sort}(\mathbf{y}) : \forall \mathbf{1} \leq \mathbf{t} \leq \mathbf{N}-\mathbf{1},\ \mathbf{y}'(\mathbf{t}) \leq \mathbf{y}'(\mathbf{t}+\mathbf{1})$
- Minimum Range Measure
  - $R(\mathbf{Y}) \approx \mathbf{y}'(\mathbf{N}) - \mathbf{y}'(\mathbf{1})$ (only 2 points are involved in est.)
  - \+ robust : $R(\mathbf{Y}) \approx \langle \mathbf{y}'(\mathbf{N}-\mathbf{q}+\mathbf{1}), \ldots, \mathbf{y}'(\mathbf{N})\rangle - \langle \mathbf{y}'(\mathbf{1}), \ldots, \mathbf{y}'(\mathbf{q})\rangle$
- Least Absolute Bound
  - $LAB(\mathbf{Y}) \approx \min\{-\mathbf{y}'(\mathbf{1}), \mathbf{y}'(\mathbf{N})\}$ (a single point is used !)
  - \+ robust: $LAB(\mathbf{Y}) \approx \min\{-\langle \mathbf{y}'(\mathbf{1}), \ldots, \mathbf{y}'(\mathbf{q})\rangle, \langle \mathbf{y}'(\mathbf{N}-\mathbf{q}+\mathbf{1}), \ldots, \mathbf{y}'(\mathbf{N})\rangle\}$
- N large enough : $\mathbf{y}'(\mathbf{q}) \approx \inf \mathbf{Y},\ \mathbf{y}'(\mathbf{N}-\mathbf{q}+\mathbf{1}) \approx \sup \mathbf{Y}$

Choosing q: [Vrins & Verleysen, ICA 2006]

Issue #2: Algo (LAB minimization)

- Mean subtraction
- Robust whitening:
  - SVD whitening directly on centered mixtures
  - Discard badly whitened mixtures keep only, let's say, $\mathbf{z_1, \ldots, z_{n-p}}$
- Deflation (for i=1:n-p)
  - LAB is minimized for directions $\mathbf{w}_i$ orthogonal to 'edges' of the joint support
    - Go away from corners
  - Random $\mathbf{w}_i^{(k=1)}$ perp to $\mathbf{w}_1, \ldots, \mathbf{w}_{i-1}$ and iterate on k ('till convergence)
    - Closest corner direction $\mathbf{u}^{(k)}$ given by normalized $\mathbf{z}$-coordinates of $\min[-\min(\mathbf{w}_i^{(k)}\mathbf{z}), +\max(\mathbf{w}_i^{(k)}\mathbf{z})]$
    - Update $\mathbf{w}_i^{(k)}$ in opposite direction (rotation towards $-\mathbf{u}^{(k)}$)
  - Slightly relaxed orthogonality constraint : almost no error accumulation ☺

If $\arccos \frac{\mathbf{w}_i^{(k)}\mathbf{w}_r}{\|\mathbf{w}_i^{(k)}\|.\|\mathbf{w}_r\|} < \pi/6$ then reinitialize

Then, put: $\mathbf{W}=[\mathbf{w}_1, \ldots, \mathbf{w}_{n-p}, \mathbf{e}_{n-p+1}, \ldots, \mathbf{e}_n]$

Algorithm: LAB minimization

- Properties of exact criterion ≈ properties of the estimated one
  - none of the minima are spurious : `convex contrast'
- Update rule: $\mathbf{w_i^{temp}} = \cos(\alpha^{(\mathbf{k})})\mathbf{w_i^{(k)}} - \sin(\alpha^{(\mathbf{k})})\mathbf{u^{(k)}}$

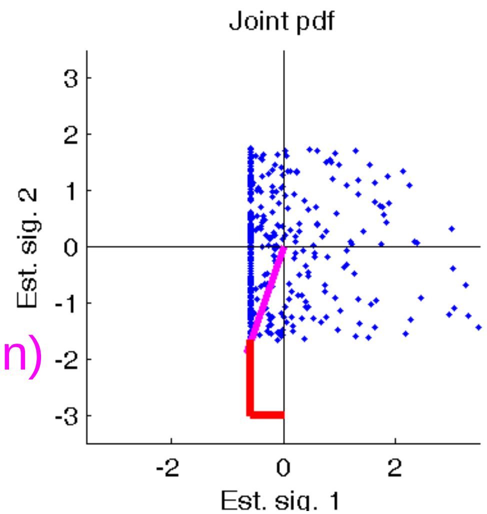
If $LAB(\mathbf{w}_i^{\text{temp}}) < LAB(\mathbf{w}_i^{(k)})$, then $\mathbf{w_i^{(k+1)}} \leftarrow \mathbf{w_i^{temp}}, \alpha^{(k+1)} \leftarrow 1.01\alpha^{(k)}$

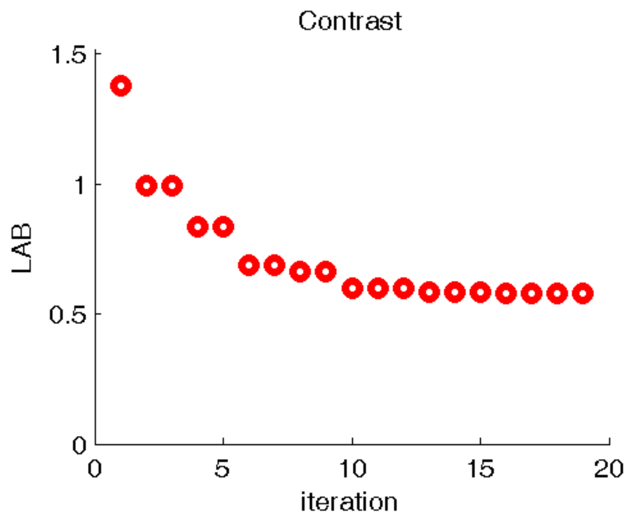
else: $\alpha^{(k+1)} \leftarrow \alpha^{(k)}/1.2$

2D demo:

LAB

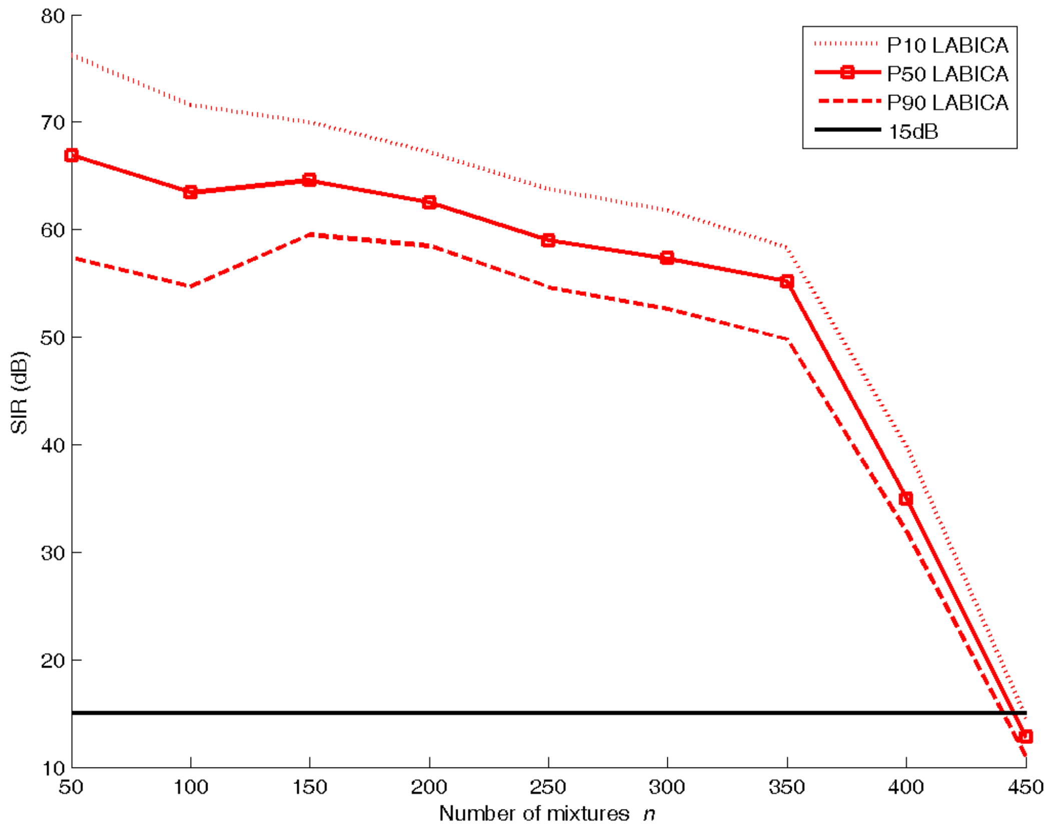
2 **u**
(corner direction)





Problem 1: large-scale mixtures

- Random mixing matrix
  - N=5000, n>50 (increasing number of sources)
- Results:
  - 20 Monte Carlo



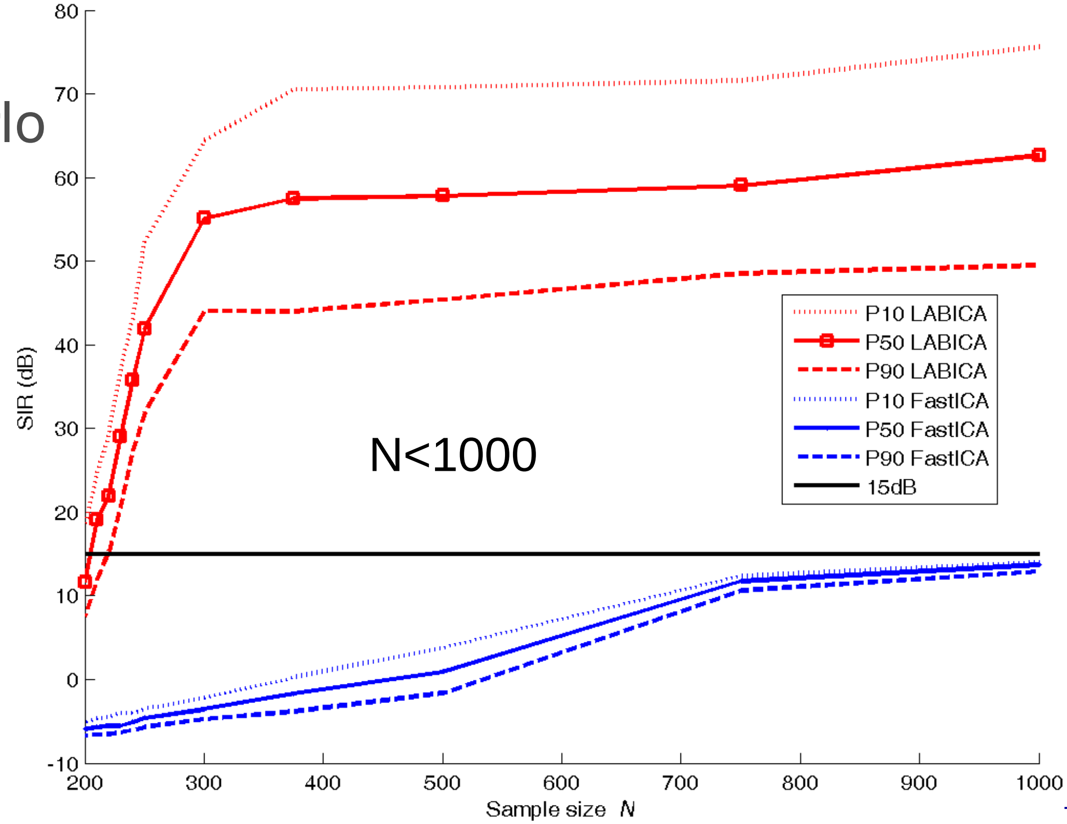
50 or 350 mixtures: almost the same separation quality ☺

More than 400 sources can be separated ☺

Problem 2: Small sample problem

- Random mixing matrix
  - N<5000 (decreasing sample size), n=50
- Results:
  - 100 Monte Carlo

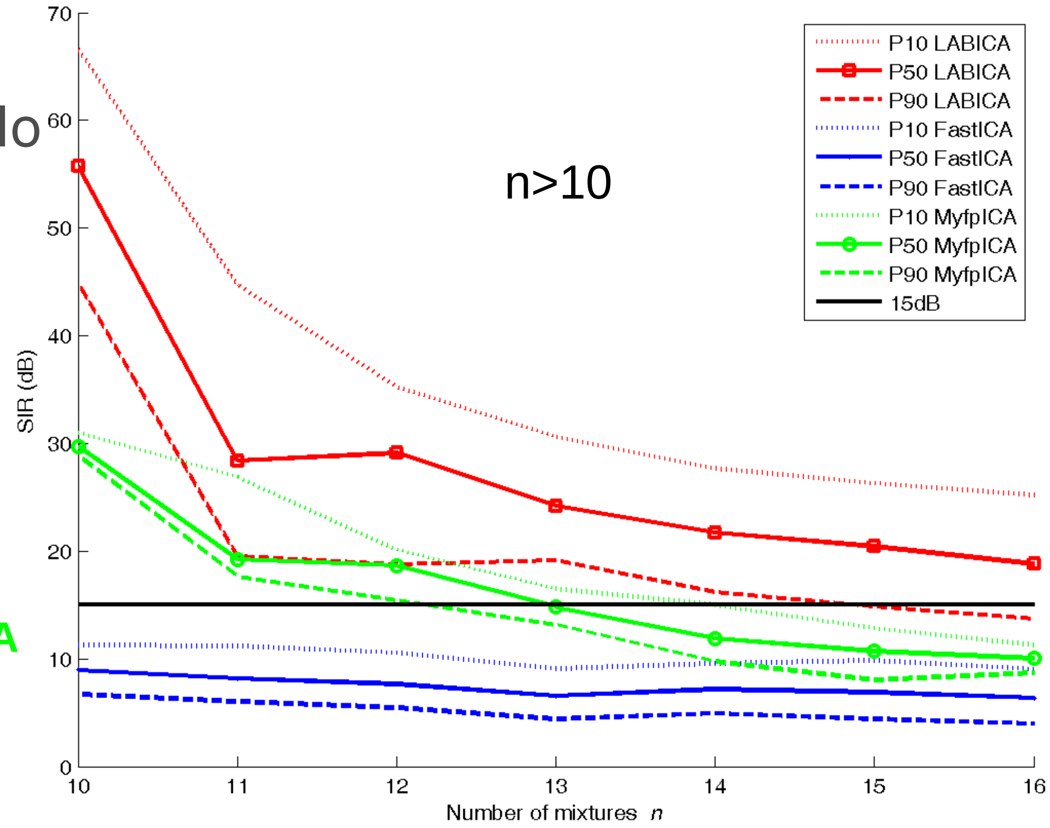
Less than 250 observations are needed to separe 50 mixtures ☺



Problem 3: Ill-conditioned Mixtures

- Mixing matrix = Hilbert times ramdom Givens
  - N=5000, n>2 (increasing number of sources)
- Results:
  - 100 Monte Carlo

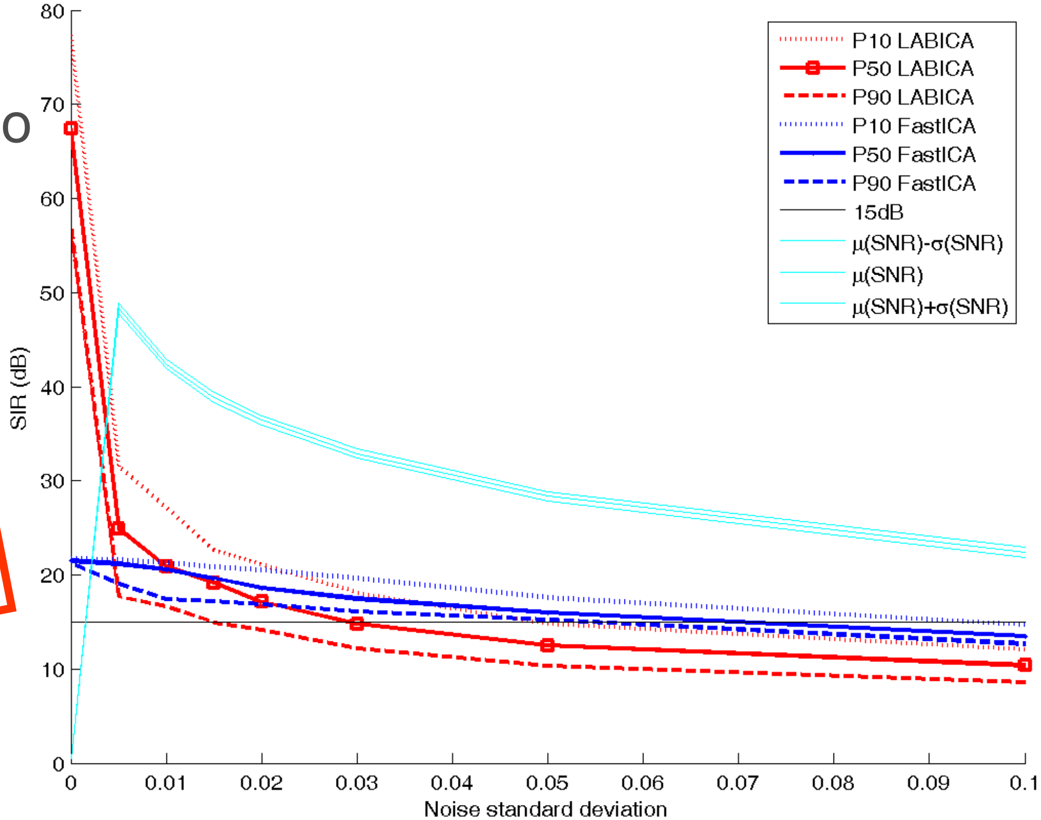
Whitening is of prime importance !!!



Myfpica = Fastica with
•Kurtosis-driven nonlin.
•Same whitening as LABICA

Problem 4: Noisy Mixtures

- Random mixing matrix + Gaussian noise
  - N=5000, n=50, increasing noise variance
- Results:
  - 100 Monte Carlo



LABICA is very sensitive to noise ☹ (pdf edges get 'blurred')

Conclusions

- Semi-blind better than blind : use the available knowledge
- Robust Whitening = important issue
  - use SVD instead of EVD
  - Smooth orthogonality constraint (avoid cumulation of errors)
- Accurate estimates of n-p srcs + bad estimates of p srcs beter than Approximative estimation of the n sources
- Non-differentiable contrasts can still efficiently be optimized
- ‘Difficult’ BSS problems can be solved because semi-blind
  - Many sources/mixtures
  - Few observations, Ill conditioned mixing matrices
  - Noise → robust method for edge estimation is a challenge
- Algo:
  - Refine and improve the optimization process (it was developed rather quickly, in the fever of the contest)

Thanks for your attention!

If you have any question, please ask…

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