Entropy Minima and Distribution Structural Modifications in Blind Separation of Multimodal Sources

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The "cocktail party" problem

Hypothesis on the mixture:

Linear and instantaneous: $\mathbf{X}(t) = \mathbf{AS}(t)$.

Blind source separation:

1. Whitening stage: $E\{\mathbf{X}\} = 0,\ E\{\mathbf{XX}^T\} = \mathbf{I}_m$.

2.Unmixing stage: $\mathbf{Y} = \mathbf{BX} = \mathbf{PD} \cdot \mathbf{S}$.

Mutual information

$$\mathcal{C}(\mathbf{Y}) = KL(f_{\mathbf{Y}} || \prod_{i=1}^m f_{Y_i}) \overset{1.}{=} \sum_{i=1}^m H(Y_i)$$

- Global minimum is acceptable solution for BSS.
- Gradient descent techniques.

Multimodal sources

Cardoso '00:

Spurious minima appear in the negative log-likelihood cost function due to a "local optimal matching" of the outputs and the target distributions.

This paper:

What happens when the cost function does not use the target distributions?

Some properties of random variables

- PDF of a weighted r.v.:

$$f_{\alpha U}(v) = \frac{1}{|\alpha|} f_U\left(\frac{v}{\alpha}\right), \alpha \in \Re$$

- PDF of a sum of r.v.:

$$f_{U+V} = f_U \otimes f_V$$

Mixture of 2 independent sources

$$\begin{cases} Y_1 &= a_{11}S_1 + a_{12}S_2 \\ Y_2 &= a_{21}S_1 + a_{22}S_2 \end{cases}$$

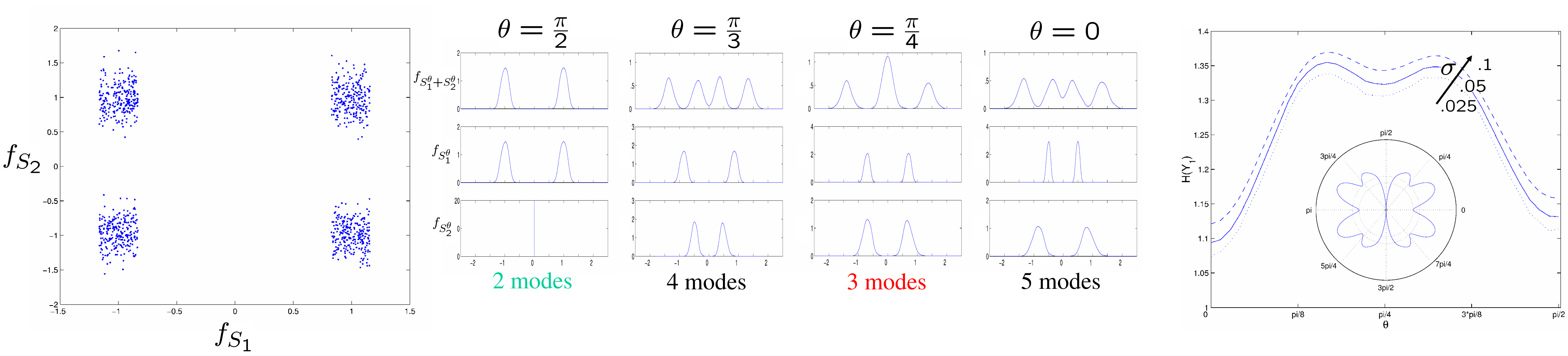
$$Y_1 = \underbrace{\sin(\theta)S_1}_{S_1^\theta} + \underbrace{\cos(\theta)S_2}_{S_2^\theta}$$

with $f_{Y_1} = f_{S_1^\theta} \otimes f_{S_2^\theta}$.

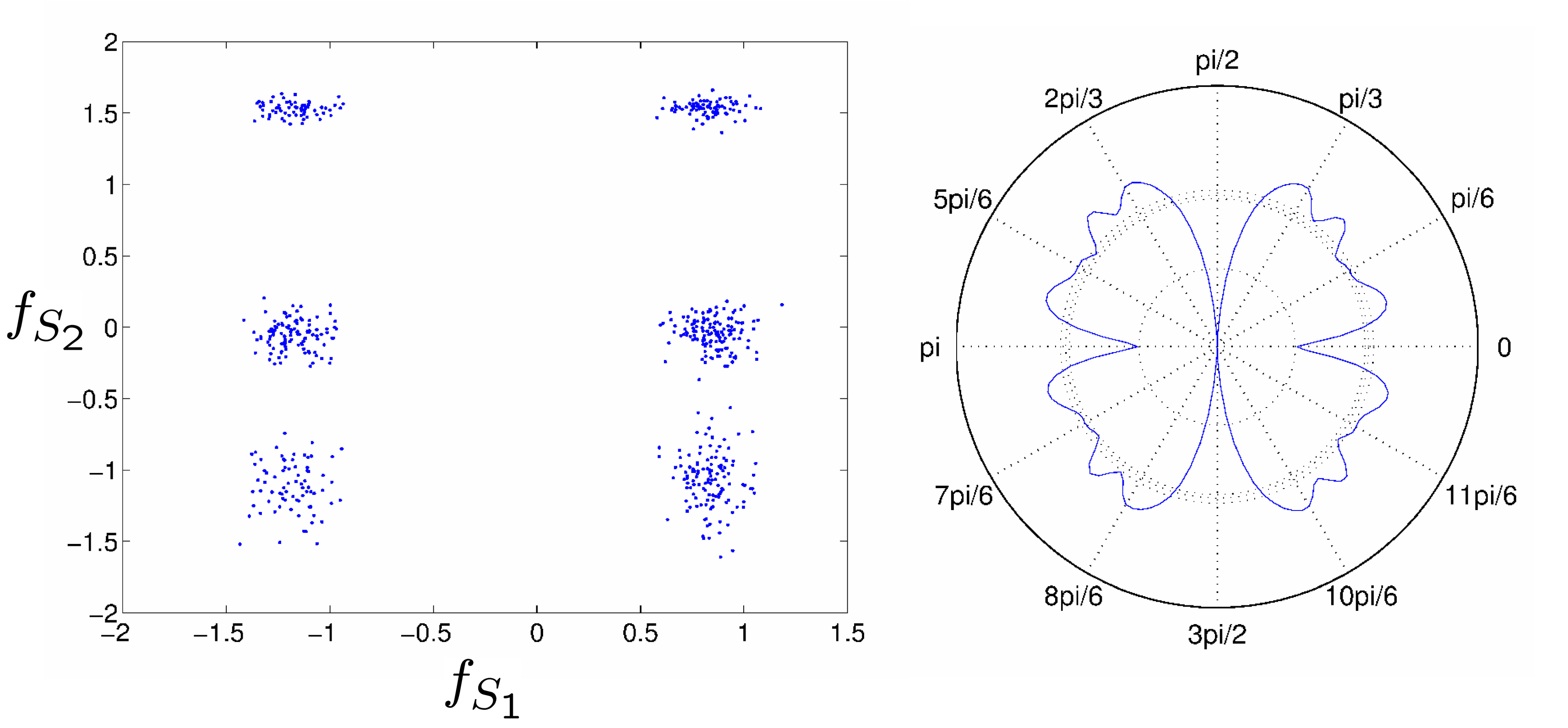
Conclusion

- Spurious minima appear when the number of modes is minimized.
- Joint effect of mixing and scaling of the independent sources while unmixing.

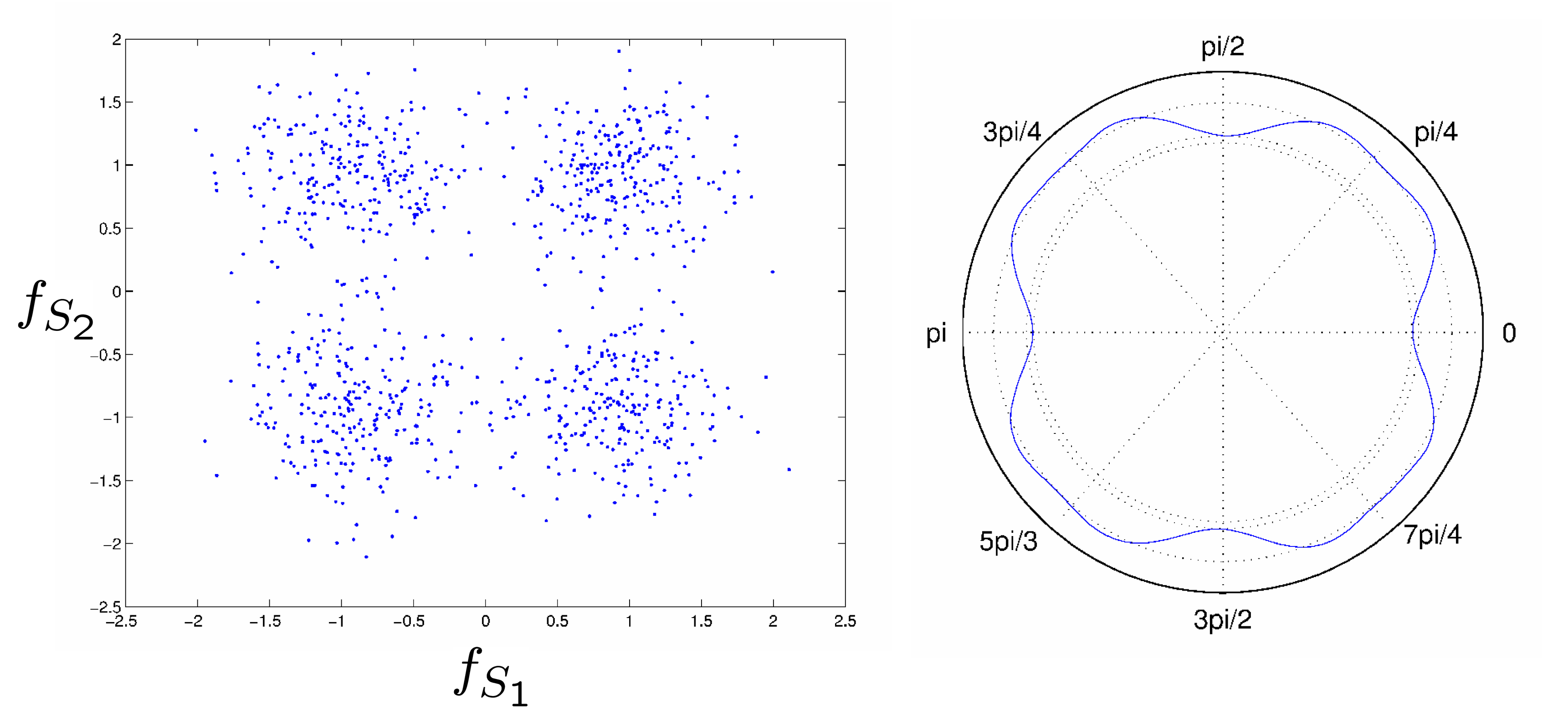
Marginal entropy



Effect of the number of modes



Effect of overlap



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